But when the semi-arch and the piers are in equilibrio with each other, the momentum of the one must be equal to that of the other. Now the weight of the pier will be appropriately represented by its area $bf \times ef$, in like manner as the weight of the semi-arch is represented by the area ACEb; and the leverage by which it resists the horizontal thrust of the semi-arch, is equal to one-half of ef; the centre of gravity of the parallelogram $abfe$ being situated in the centre of magnitude; consequently, the momentum of the pier to resist the horizontal thrust of the semi-arch, is $\frac{1}{2}ef^2 \cdot bf$: therefore, by equating this with the momentum of the semi-arch, we get

$$\tfrac{1}{2}ef^2 \cdot bf = \frac{w \cdot kA \cdot Af}{Gk}.$$

This equation is general, and will assume the same form whatever may be the nature of the curve on which the arch is formed. Reducing it in reference to ef, the thickness of the pier, we get

$$ef = \sqrt{\frac{2w \cdot kA \cdot Af}{bf \cdot Gk}} \;\ldots\ldots (I)$$

In this equation the quantities Gk in the denominator of the fraction, and kA in the numerator are unknown, and it is these that we now proceed to determine.

In consequence of the very simple mensuration of the parabola, the position of the centre of gravity at G is easily found; for we have only to consider the rectangular parallelogram ADEb, and the parabolic space ACD, which is in the same plane with it, and forming a part of itself, to revolve about the semi-base AD, which remains fixed, in this case, the parallelogram will generate a cylinder of which the radius is DE, and altitude AD, while the parabolic space describes a conoidal solid, the radius of whose base is DC and altitude or axis AD; it therefore follows, that the difference between these two solids is equal to the solid generated by the figure ACEb during the revolution.

By the rules of mensuration, the solidity of the generated cylinder is expressed by the term $3{\cdot}1416 \mathrm{AD} \cdot \mathrm{DE}^2$, and that of the conoidal figure by the term $1{\cdot}67552 \mathrm{AD} \cdot \mathrm{DC}^2$; and the difference of these is

$$3{\cdot}1416\mathrm{AD}\cdot\mathrm{DE}^2 - 1{\cdot}67552\mathrm{AD}\cdot\mathrm{DC}^2 = 1{\cdot}67552\mathrm{AD}(1{\cdot}875\mathrm{DE}^2 - \mathrm{DC}^2)$$

This is the solidity of the figure which is generated by the revolution of the semi-arch ACEb; but it is evident, that while the semi-arch revolves about AD, and describes the solid just named, the centre of gravity G describes a circle whose radius is kG; and by the properties of Guldinus, the solidity of the figure described by the revolution of the plane ACEb, is equal to that of a prism whose base is the area of the describing plane, and altitude equal to the circumference of the circle traced out by the centre of gravity of that plane; consequently, by having both the area of the semi-arch, and the solidity of the figure generated by that area, the position of the centre of gravity, in reference to the semi-base AD, can readily be found. This is the principle to which the reader's attention is more particularly directed; for by its application, the position of the centre of gravity of the balancing materials of an arch can be more readily found than by any other method whatever, since it requires no other knowledge than the simple rules of mensuration.

The area of the semi-arch ACEb, is equal to the difference between the rectangle ADEb and the semi-parabola ACD; but the area of the rectangle is expressed by the term AD × DE, and that of the semi-parabola by $\frac{2}{3}$AD·DC; and the difference of these is $\mathrm{AD}\cdot\mathrm{DE} - \frac{2}{3}\mathrm{AD}\cdot\mathrm{DC} = \mathrm{AD}\,(\mathrm{DE} - \frac{2}{3}\mathrm{DC})$, and it has been shewn that the solidity of the figure generated by that area, is $1{\cdot}67552\mathrm{AD}\,(1{\cdot}875\mathrm{DE}^2 - \mathrm{DC}^2)$; therefore by division, we get

$$\frac{1{\cdot}67552\mathrm{AD}\,(1{\cdot}875\mathrm{DE}^2 - \mathrm{DC}^2)}{\frac{1}{3}\mathrm{AD}\,(3\mathrm{DE} - 2\mathrm{DC})} = \frac{5{\cdot}02656\,(1{\cdot}875\mathrm{DE}^2 - \mathrm{DC}^2)}{(3\mathrm{DE} - 2\mathrm{DC})}$$

This is the circumference of the circle described by the centre of gravity of the semi-arch, and the radius will be found by dividing by 6·2832; hence we have

$$\textit{co-ordinate},\ k\mathrm{G} = \frac{0{\cdot}8\,(1{\cdot}875\mathrm{DE}^2 - \mathrm{DC}^2)}{(3\mathrm{DE} - 2\mathrm{DC})} \ldots (\mathrm{K})$$

This equation expresses the value of the vertical distance between the base or span of the arch, and the centre of gravity of the semi-arch at G, and by a similar process the horizontal co-ordinate, or the distance between the axis and the same centre of gravity, can readily be found. For let us now suppose, that the rectangular plane ADEb, and the semi-parabola ACD, perform their revolutions simultaneously about the axis DE, which remains fixed, the one will generate a cylinder whose radius is AD, and altitude DE, while the other generates a paraboloid, whose radius is AD and axis DC. But by the rules of mensuration, the solidity of the cylinder is represented by the term $3{\cdot}1416\mathrm{AD}^2\cdot\mathrm{DE}$, while that of the paraboloid is represented by $1{\cdot}5708\mathrm{AD}^2\cdot\mathrm{DC}$, and the difference of these is

$$3{\cdot}1416\mathrm{AD}^2\cdot\mathrm{DE} - 1{\cdot}5708\mathrm{AD}^2\cdot\mathrm{DC} = 1{\cdot}5708\mathrm{AD}^2\,(2\mathrm{DE} - \mathrm{DC}).$$

This is the solidity of the figure described by the semi-arch revolving about the axis DE, and we have already seen, that the area of the describing plane, is represented by $\frac{1}{3}$AD (3DE − 2DC); therefore, by division we get

$$\frac{1{\cdot}5708\mathrm{AD}^2(2\mathrm{DE} - \mathrm{DC})}{\frac{1}{3}\mathrm{AD}(3\mathrm{DE} - 2\mathrm{DC})} = \frac{4{\cdot}7124\mathrm{AD}(2\mathrm{DE} - \mathrm{DC})}{(3\mathrm{DE} - 2\mathrm{DC})}.$$

And this is the circumference of the circle described by the centre of gravity of the semi-arch while it revolves about the axis DE. Let this be again divided by the constant number 6·2832, and we obtain for the

$$\textit{co-ordinate},\ \mathrm{GH} = \frac{{\cdot}75\mathrm{AD}\,(2\mathrm{DE} - \mathrm{DC})}{(3\mathrm{DE} - 2\mathrm{DC})},$$

and if this be subtracted from the semi-base AD, it becomes

$$i\mathrm{G} = \mathrm{AD} - \frac{{\cdot}75\mathrm{AD}(2\mathrm{DE} - \mathrm{DC})}{(3\mathrm{DE} - 2\mathrm{DC})}.$$

which, being reduced to its simplest form, gives

$$i\mathrm{G} = k\mathrm{A} = \frac{\mathrm{AD}(1{\cdot}5\mathrm{DE} - 1{\cdot}25\mathrm{DC})}{(3\mathrm{DE} - 2\mathrm{DC})} \ldots (\mathrm{L})$$

Now, if these values of kA and kG, that is, the horizontal and vertical co-ordinates, be substituted instead of them in the equation marked (I), we shall obtain

$$ef = \sqrt{\frac{2w \cdot \mathrm{A}f \cdot \mathrm{AD}(1{\cdot}5\mathrm{DE} - 1{\cdot}25\mathrm{DC})}{bf\,(1{\cdot}5\mathrm{DE}^2 - 0{\cdot}8\mathrm{DC}^2)}},$$

And, finally, by substituting $2\mathrm{AD}(\mathrm{DE} - \frac{2}{3}\mathrm{DC})$, or twice the area of the semi-arch, for $2w$, the expression for the breadth or thickness of the pier, becomes

$$ef = \sqrt{\frac{2\mathrm{A}f\,\mathrm{AD}^2(1{\cdot}5\mathrm{DE} - 1{\cdot}25\mathrm{DC})(3\mathrm{DE} - 2\mathrm{DC})}{bf\,(4{\cdot}5\mathrm{DE}^2 - 2{\cdot}4\mathrm{DC}^2)}} \ldots (\mathrm{M})$$

In this equation there is nothing but known quantities, all having reference to the dimensions of the arch and its supporting piers; it is, however, of such a form as not to admit of being clearly expressed in words, and consequently the method of reducing it must be obtained by an attentive examination of the several steps in the working of the following example:—

Example 3.—Suppose the arch ACB to be a parabola, of which the span or base AB is 40 feet, the axis or height DC 20 feet, the thickness at the crown CE 4 feet, and fA, the distance between the bottom of the pier at f and impost at A 8 feet; what must be the thickness of the pier to sustain the thrust of the arch, the summit of the pier and the roadway being in the same horizontal plane?

Here, then, we have given Af = 8 feet; AD = 20 feet; DC = 20 feet; DE = 20 + 4 = 24 feet, and bf = 20 + 4 + 8 = 32 feet. Hence we get, $1{\cdot}5\mathrm{DE} - 1{\cdot}25\mathrm{DC} = 24 \times 1{\cdot}5 - 20 \times 1{\cdot}25 = 36 - 25 = 11$;

$3\mathrm{DE} - 2\mathrm{DC} = 24 \times 3 - 20 \times 2 = 72 - 40 = 32$;
$2\mathrm{A}f \times \mathrm{AD}^2 = 8 \times 2 \times 20 \times 20 = 16 \times 400 = 6400$;

Then, if these three results be multiplied together, we get 11 × 32 × 6400 = 2252800; this is the numerator of the fraction under the vinculum in equation (M), and the denominator is found as follows:—

$$4{\cdot}5\mathrm{DE}^2 - 2{\cdot}4\mathrm{DC}^2 = 4{\cdot}5 \times 24 \times 24 - 2{\cdot}4 \times 20 \times 20 = 2592 - 960 = 1632,$$

which being multiplied by bf = 32, we get, 1632 × 32 = 52224 for the denominator of the fraction; therefore by dividing and extracting the square root, we obtain

$$ef = \sqrt{\frac{2252800}{52224}} = \sqrt{43{\cdot}1372} = 6{\cdot}57 \text{ feet}$$

very nearly.

N. B.—This form of arch is very frequently employed in the construction of powder magazines and other military works. T.

DWELLINGS, DRAINS, AND SUPPLY OF WATER IN THE CITY.

On the 12th inst., Dr. Lynch presented to a Court of Common Council a well-timed and judicious petition on subjects of the utmost importance, as affecting the health and well-being of the community, and supported the prayer of it in an admirable speech, which ought to be published in a separate form and distributed throughout the country.

We purposely avoided reference to the statement at the moment; when it occupied the attention of the daily papers, in order by recurrence, to increase its effect. The subject being of permanent and engrossing interest, we feel impelled to record the petition in full:—

"*To the Right Hon. the Lord Mayor, the Aldermen and Commonalty of the City of London, in Common Council Assembled.*

The following petition of the Metropolitan Working Classes Association for Improving the public health.

Humbly sheweth—That the corporation have set the example of applying a large, and as your petitioners are informed, a sum of no less than 20,000*l.* per annum, to the improvement of the city as a part of the metropolis.

That your petitioners think the improvements good in themselves; that the widening of the streets is good as a means of ventilation; that your petitioners admire wide and magnificent streets, and take pride in the improved general aspect of the city and of the metropolis. But they humbly submit, that whilst all is thus made fair and good to behold without, all should not be left foul and miserable within; and that these benefits may be carried out otherwise than to the injury of the working classes, which effect has been and is now produced.

Your petitioners aver, that by every old street occupied by the labouring classes pulled down to form a new street, the evil of overcrowding the existing old streets, courts, and tenements near the street pulled down, is generally aggravated.

Your petitioners know that this effect was produced by the pulling down of houses to form the Farringdon-market. They know (and it is in evidence before the Commissioners for Inquiring into the Means of Improving the Health of Towns) that the like effect was produced by the pulling down old houses to form the Blackwall Railway. They know that the like effect has been produced by the removal of the old and poor houses to form the new streets which traverse the site of St. Giles's. Good, new streets have been formed, but the poor population of St. Giles's has been only wedged in more densely upon the poor population of Seven Dials, and the bad made worse.

Your petitioners see that new lines of railway are proposed to be carried, some into the city of London, others through other parts of the metropolis, and that large numbers of old houses are to be pulled down, but they see no provision made, or consideration given, for building new ones for the accommodation of the poor people who cannot remove from their places of work.

Your petitioners pray that whilst the corporation devotes its funds to the formation of new and magnificent houses, it will at the same time exercise its power in building for the working classes, at reasonable and yet remunerative rents, improved tenements, in numbers equal to those pulled down, and by so doing a benefit will be conferred, instead of an injury inflicted, on those whose condition most need improvement.

Your petitioners observe with much satisfaction that new large main sewers are in course of construction down the main streets. They regret, however, to find that the streets occupied by the labouring classes are still badly drained; that their houses are still left undrained; and whilst in the new prisons, and for the use of prisoners, an apparatus of the nature of a water-closet is provided, and a supply of water carried into each cell, the houses of the working classes are only provided with disgusting and unwholesome privies and cesspools, and have only dear and bad supplies of water, kept stagnant in butts in the yards, exposed to contamination from soot and dust.

Your petitioners further pray, that at the same time that drains for houses and sewers for streets are provided, your hon. Court will exercise its power and influence to procure a good constant supply of water to make the drains and sewers to act, and constantly remove all accumulations of offensive matter, and to have such supplies carried at a cheap rate into every set of rooms occupied by honest and industrious labouring men, who are willing to pay a fair and reasonable rate for the accommodation; whereas the price now charged for bad supplies, is proved to be more than three times the amount for which supplies of a better description might be ensured.